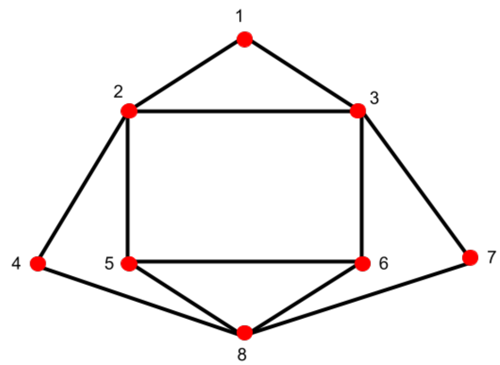
2022 Accuracy Round Solutions

Key (solutions start on next page)

1. 6
2. 34
3. 172
4. 4
5. 21
6. 16
7. 127
8. 10
9. 135
10. 17

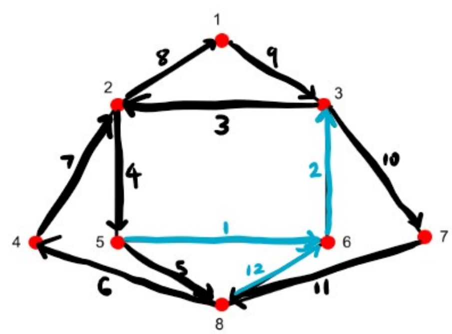
Solutions

1. Mikhail puts his pencil at point 5 in the following picture and traces over the lines in such a way that he traces each line exactly once and never picks up his pencil. Which labeled red point (1, 2, 3, 4, 5, 6, 7 or 8) does his pencil end at?



Answer: $\boxed{6}$.

Solution: There are 3 lines which have an endpoint at point 6. The first time Mikhail traces one of those 3 lines, he must be going into point 6. The second time Mikhail traces one of those lines, he must be going out of point 6. The third and final time Mikhail traces one of those lines, he must be going into point 6. There are no more ways left to get out of point 6, so this must be the final point, i.e. Mikhail must end at point 6.



□

2. The news forecasts a 70% chance of rain on Saturday. If it rains on Saturday, there is a 10% chance of it raining on Sunday. Otherwise, if it doesn't rain on Saturday, there is a 90% chance of it raining on Sunday. What is the percent chance that it rains on Sunday?

Answer: $\boxed{34}$.

Solution: There are two possibilities we must consider:

- It rains on Saturday, and then it rains again on Sunday. The total probability that this happens is $(70\%)(10\%) = 7\%$.
- It doesn't rain on Saturday, and then it rains on Sunday. The total probability that this happens is $(30\%)(90\%) = 27\%$.

This means that overall, there is a $7\% + 27\% = 34\%$ chance that it rains on Sunday. □

3. Carolina writes the numbers $1, 2, 3, \ldots, 100$ on a whiteboard. Then Scott comes and erases all instances of the digit "2" on the whiteboard (e.g. when he sees the number 21, he erases the 2 and leaves the 1). How many digits remain on the whiteboard?

Answer: $\boxed{172}$.

Solution: Initially, there were 9 one-digit-numbers, 90 two-digit-numbers, and 1 three-digit-number. This is a total of $9 \cdot 1 + 90 \cdot 2 + 1 \cdot 3 = 192$ digits.

- There are 10 numbers that have 2 as the units digit: $2, 12, 22, \ldots, 92$. Hence Scott erases 10 units-digit-2's.
- There are 10 numbers that have 2 as the tens digit: $20, 21, 22 \ldots, 29$. Hence Scott erases 20 tens-digit-2's.

This means Scott erased $10 + 10 = 20$ digits in total, so the number of digits remaining is $192 - 20 = 172$. □

4. How many even positive integers are divisors of the number 2022, which has prime factorization $2 \times 3 \times 337$?

Answer: $\boxed{4}$.

Solution: Since $2, 3$, and 337 are prime, any divisor of $2 \times 3 \times 337$ must be a product of some of the numbers $2, 3$ and 337. Moreover, 2 must be included in that product, since the divisor must be even. There are four such numbers: $2, 2 \times 3, 2 \times 337$, and $2 \times 3 \times 337$. □

5. Isaac has 5 pennies, 5 nickels, and 5 dimes. He randomly takes 5 coins without looking. How many different possible total amounts of money could he have taken?

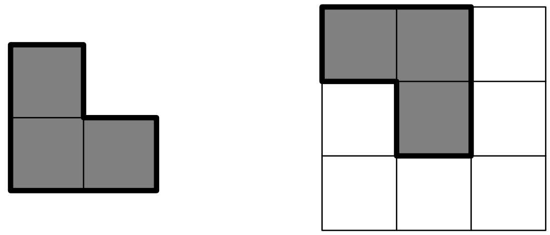
Answer: $\boxed{21}$.

Solution: We have 6 distinct cases, based on the number of pennies we have:

- If we have 0 pennies, then we can have anywhere between 0 and 5 dimes, and the rest of the coins are nickels. This corresponds to 6 possibilities.
- If we have 1 penny, then we can have anywhere between 0 and 4 dimes, and the rest of the coins are nickels. This corresponds to 5 possibilities.
- Similarly, if we have 2 pennies, then there are 4 possibilities.
- If we have 3 pennies, there are 3 possibilities.
- If we have 4 pennies, there are 2 possibilities.
- If we have 5 pennies, there is 1 possibility.

In total, there are $6 + 5 + 4 + 3 + 2 + 1 = 21$ possible amounts of money Isaac could have. □

6. An L-shaped version of a domino with 3 cells, as shown below, is known as a triomino. How many ways are there to place a triomino in a 3x3 grid, such that the cells of the triomino line up with the grid lines? One of the ways is shown below (the triomino may be rotated).



Answer: $\boxed{16}$.

Solution: There are four possible orientations of the triomino. For each of these four orientations, we can see that there are four ways to place it in the grid. Hence the total number of ways to place the triomino in the grid is $4 \cdot 4 = 16$. □

7. 64 students play in a double elimination ping pong tournament. Each game ends in a win for one player and a loss for the other (there are no draws). Once a player has lost twice, they are eliminated. What is the maximum number of games that can be played before only one winner remains?

Answer: $\boxed{127}$.

Solution: The total number of games played has to equal the total number of losses over all the players. If 63 players have been eliminated, each of them must have lost twice. The remaining player is either undefeated, or has lost once. Hence the total number of losses is either $63 \cdot 2$ or $63 \cdot 2 + 1$. Since we're looking for the largest possible answer, the answer is $63 \cdot 2 + 1 = 127$. □

8. We say two different positive integers are *besties* if the larger one leaves a remainder of 2 when divided by the smaller one. How many ordered triples of three distinct positive integers (a, b, c) where $3 \leq a, b, c \leq 9$ are there such that a and b are besties, and b and c are besties? For example, $(3, 5, 7)$ and $(7, 5, 3)$ are two different ordered triples that work.

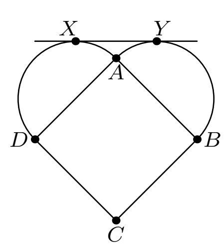
Answer: $\boxed{10}$.

Solution: Draw the following diagram, where two numbers are connected by a line if they are besties.

$$4 — 6 — 8 — 3 — 5 — 7 — 9$$

We can see that b must be one of the 5 numbers in the middle, so there are 5 choices for b. Once we pick b, we can pick either of it's neighbors to be a, and then the other neighbor must be c. Hence the total number of ways is $5 \cdot 2 = 10$. □

9. Square $ABCD$ has semicircles drawn outside it with diameters AB and AD. We then draw a line parallel to line BD, and tangent to both semicircles at points X and Y, as shown. What is the measure of $\angle XAY$ in degrees?



Answer: $\boxed{135}$.

Solution: Let P, Q be the centers of the semicircles with diameters AD and AB, respectively. Note that PX and PY are both perpendicular to XY. Consider $\triangle PXA$: it is isosceles with $PA = PX$, because they are both radii of the semicircle. This means that $\angle XAP = \angle AXP$. In addition, notice that $\angle XPA = 45°$, since line XP is vertical. Since the angles in a triangle sum to $180°$, this means

$$180° = \angle AXP + \angle XAP + \angle XPA = 2\angle XAP + 45° \implies \angle XAP = 67.5°.$$

By symmetry, we also have $\angle YAQ = 67.5°$. Now we can compute that

$$\angle XAY = 360° - \angle XAP - \angle YAQ - \angle PAQ = 360° - \angle 67.5° - \angle 67.5° - 90° = 135°.$$

□

10. The polynomial $x^3 - 7x^2 + 3x + 1$ can be written as a product of factors $(x-a)(x-b)(x-c)$ for some real numbers a, b, c. What is the value of the expression $(1-a-b)(1-b-c)(1-a-c)$?

Answer: $\boxed{17}$.

Solution: By expanding the product $(x-a)(x-b)(x-c)$ and setting it equal to $x^3 - 7x^2 + 3x + 1$, we get that $a + b + c = 7$.[1] This means that $1 - a - b = c - 6$, etc., so the quantity we want to compute is equal to

$$(a-6)(b-6)(c-6) = -(6-a)(6-b)(6-c).$$

However, we're given that $(x-a)(x-b)(x-c) = x^3 - 7x^2 + 3x + 1$. Plugging in $x = 6$, this tells us that $(6-a)(6-b)(6-c) = 6^3 - 7 \cdot 6^2 + 3 \cdot 6 + 1 = -17$. Hence the answer we want is 17. □

[1] see https://en.wikipedia.org/wiki/Vieta%27s_formulas